1(a) We want to solve

$$1+\frac{dy}{dx}e^{3x}=0$$

We use the "informal method". See the class notes if you want to formalize this without differentials.

We have

$$\frac{dy}{dx}e^{3x}=-1$$

So,

$$dy=-e^{-3x}dx$$

Thus,

$$\int 1\cdot dy=-\int e^{-3x}dx$$

So,

$$y=-\left(-\frac{1}{3}e^{-3x}\right)+C$$

Thus,

$$y=\frac{1}{3}e^{-3x}+C$$

where C is any constant.

Note that this function is defined on $I=(-\infty,\infty)$

1 (b) In part (a) we saw that

$$y = \frac{1}{3}e^{-3x} + C$$

is a solution to

$$1 + \frac{dy}{dx}e^{3x} = 0$$

We also want $y(0) = -5$.

Plugging in $x=0$, $y=-5$ into the above solution we have

$$-5 = \frac{1}{3}\underbrace{e^{-3(0)}}_{e^0=1} + C$$

So,

$$-5 = \frac{1}{3} + C$$

Thus,

$$C = -5 - \frac{1}{3} = \frac{-15-1}{3} = \frac{-16}{3}$$

So,

$$y = \frac{1}{3}e^{-3x} - \frac{16}{3}$$

is a solution to

$$1 + \frac{dy}{dx}e^{3x} = 0, \quad y(0) = -5.$$

1 (c) Want to solve $\frac{dy}{dx} = -\frac{x}{y}$

We will use the "informal method" with differentials. See the class notes if you want to use a more formal method without differentials.

We have that

$$\frac{dy}{dx} = \frac{-x}{y}$$

Separating the x's and y's gives

$$y\,dy = -x\,dx$$

Thus,

$$\int y\,dy = -\int x\,dx$$

So,

$$\frac{y^2}{2} = -\frac{x^2}{2} + C$$

Thus,

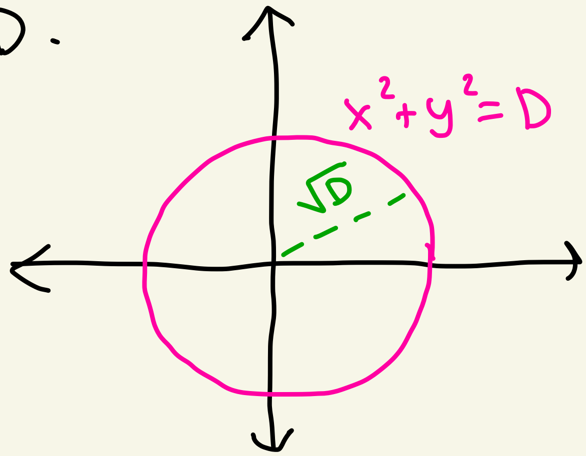
$$y^2 = -x^2 + 2C$$

So, $x^2 + y^2 = D$

where $D = 2C$ is any constant

See next page for more info

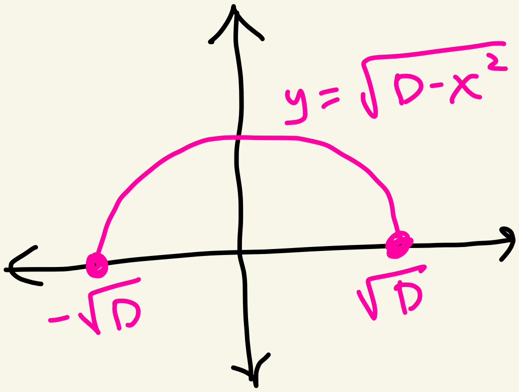
Note that the solution has the form $x^2 + y^2 = D$. So it's a circle with radius $\sqrt{D}$.



If you want a function for y then you'd need to take square roots and you'd get either

Solution 1

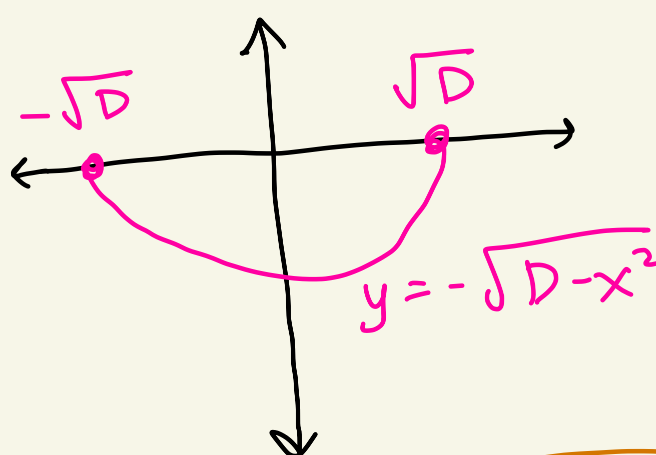
$$y = \sqrt{D - x^2}$$



OR

Solution 2

$$y = -\sqrt{D - x^2}$$



both are defined on $[-\sqrt{D}, \sqrt{D}]$.

1(d) We saw in part (c) that a solution to

$$\frac{dy}{dx} = -\frac{x}{y}$$

is

$$x^2 + y^2 = D.$$

We want $y(4)=3$. Plug in $x=4, y=3$ into our solution to get:

$$4^2 + 3^2 = D$$

So,

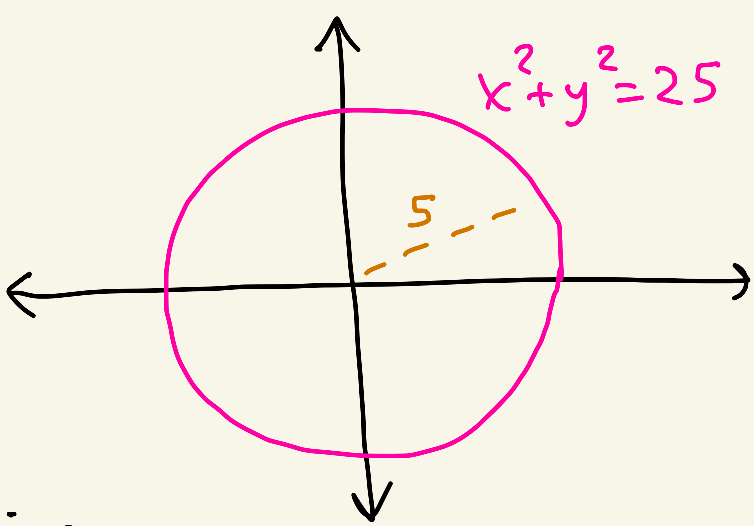
$$D = 25$$

Thus, an implicit solution to

$$\frac{dy}{dx} = \frac{-x}{y}, \quad y(4)=3$$

is

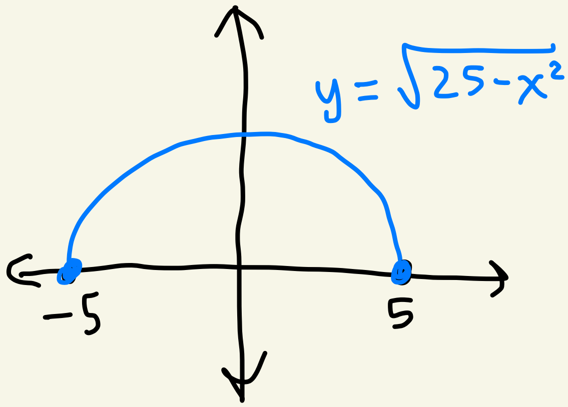
$$\boxed{x^2 + y^2 = 25}$$



If you solve for y then you would get two functions ↗

Function 1

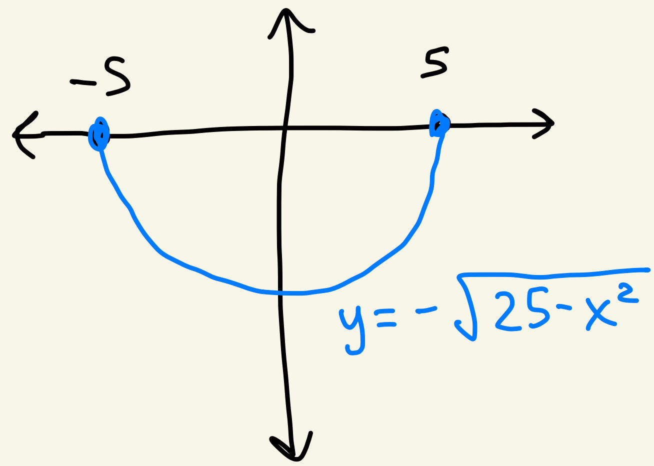
$$y=\sqrt{25-x^2}$$



OR

Function 2

$$y=-\sqrt{25-x^2}$$



Both of these functions are defined on $I=[-5,5]$.

However only function 1 satisfies $y(4)=3$

Thus, an answer to the problem is

$$\boxed{y=\sqrt{25-x^2}}$$

$\boxed{1(e)}$ We want to solve

$$x e^{-y} \sin(x) - y \frac{dy}{dx} = 0$$

We use the "informal method". See the class notes if you want to formalize this without differentials.

We have

$$y \frac{dy}{dx} = x e^{-y} \sin(x)$$

So,

$$y e^{y} dy = x \sin(x)\, dx$$

Thus,

$$\int y e^{y} dy = \int x \sin(x)\, dx$$

Note:

$$\int y e^{y} dy = y e^{y} - \int e^{y} dy = y e^{y} - e^{y} + C_1$$

$u = y \qquad du = dy$

$dv = e^{y} dy \qquad v = e^{y}$

$\int u\,dv = uv - \int v\,du$

$$\int x \sin(x)\, dx = -x\cos(x) + \int \cos(x)\, dx = -x\cos(x) + \sin(x) + C_2$$

$u = x \qquad du = dx$

$dv = \sin(x) \qquad v = -\cos(x)$

Thus we get

$$\boxed{ye^y - e^y = -x\cos(x) + \sin(x) + C}$$

constants of integration C_1, C_2 combined $C = C_2 - C_1$

Thus we get an implicit equation relating y and x but we can't really solve for y.

1(f) In part (e) we saw that

$$ye^y - e^y = -x\cos(x) + \sin(x) + C$$

gave an implicit solution to

$$xe^{-y}\sin(x) - y\frac{dy}{dx} = 0$$

We also want $y(0)=1$.
Plug in $x=0, y=1$ into our solution to get

$$\underbrace{1\cdot e^1 - e^1}_{0} = \underbrace{-0\cdot\cos(0)}_{0} + \underbrace{\sin(0)}_{0} + C$$

So,

$$C = 0.$$

Thus,

$$ye^y - e^y = -x\cos(x) + \sin(x)$$

gives an implicit solution to

$$xe^{-y}\sin(x) - y\frac{dy}{dx} = 0, \quad y(0) = 1$$

$\boxed{①(g)}$ We want to solve

$$xy' = 4y$$

We use the "informal method". See the class notes if you want to formalize this without differentials.

We have

$$x \cdot \frac{dy}{dx} = 4y$$

Thus,

$$\frac{dy}{4y} = \frac{dx}{x}$$

So,

$$\frac{1}{4}\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

Thus,

$$\frac{1}{4}\ln|y| = \ln|x| + C$$

So,

$$\ln|y| = 4\ln|x| + 4C$$

Let's use e to get rid of the ln's.

We get

$$e^{\ln|y|} = e^{4\ln|x| + 4C}$$

So,

$$e^{\ln|y|} = e^{\ln(|x|^4)+4C}$$

$A\ln(B) = \ln(B^A)$

Thus,

$$e^{\ln|y|} = e^{\ln(|x|^4)} \cdot e^{4C}$$

$e^{\ln(A)} = A$

So,

$$|y| = |x|^4 \cdot e^{4C}$$

Thus,

$$|y| = D \cdot |x|^4$$

where $D = e^{4C} > 0$ is a positive constant.

So,

$$y = \pm D \cdot x^4$$

$$\boxed{\begin{array}{l}\text{Thus,}\\ \quad y = A \cdot x^4\\ \text{where } A \text{ is any constant.}\\ \text{This solution is defined}\\ \text{on } I = (-\infty, \infty)\end{array}}$$

1 (h) In part (g) we saw that

$y = Ax^4$ is a solution to $xy' = 4y$.

To get $y(1) = 5$ we plug $x=1, y=5$ into our solution to get

$$5 = A \cdot (1)^4$$

So,

$$A = 5$$

Thus,

$$y = 5x^4$$

solves

$$xy' = 4y, \quad y(1) = 5$$